

$$\frac{a_{u,2}/B'_{0M}/\Sigma \epsilon \lambda 48}{}$$

$$(i) \quad |\vec{u} + \vec{v}|^2 + |\vec{u} - \vec{v}|^2 = 2|\vec{u}|^2 + 2|\vec{v}|^2.$$

$$\text{d' Héraults: } |\vec{u} + \vec{v}|^2 + |\vec{u} - \vec{v}|^2 = (\vec{u} + \vec{v})^2 + (\vec{u} - \vec{v})^2 =$$

$$= \vec{u}^2 + \cancel{2\vec{u}\vec{v}} + \vec{v}^2 + \vec{u}^2 - \cancel{2\vec{u}\vec{v}} + \vec{v}^2 = 2\vec{u}^2 + 2\vec{v}^2 = 2|\vec{u}|^2 + 2|\vec{v}|^2 \quad \text{O.E.S.}$$

$$(ii) \quad \vec{u} \cdot \vec{v} = \frac{1}{4} |\vec{u} + \vec{v}|^2 - \frac{1}{4} |\vec{u} - \vec{v}|^2$$

$$\text{B' Héraults} = \frac{1}{4} (\vec{u} + \vec{v})^2 - \frac{1}{4} (\vec{u} - \vec{v})^2 = \left(\frac{1}{4} (u^2 + 2\vec{u}\vec{v} + v^2) \right) - \left(\frac{1}{4} (u^2 - 2\vec{u}\vec{v} + v^2) \right)$$

$$= \frac{1}{4} (\vec{u}^2 + \cancel{2\vec{u}\vec{v}} + \vec{v}^2 + \cancel{2\vec{u}\vec{v}} + \vec{v}^2) = \frac{1}{4} 4\vec{u}\vec{v} = \vec{u}\vec{v} \quad \text{O.E.S.}$$

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$$|\vec{a}| = 2, |\vec{b}| = 1, |\vec{\gamma}| = 3, 2\vec{a} + \vec{b} + \vec{\gamma} = \vec{0} \quad (1)$$

$$(i) \quad \underline{\vec{a} \cdot \vec{b} = ?}$$

$$(1) \Leftrightarrow 2\vec{a} + \vec{b} = -\vec{\gamma} \Rightarrow$$

$$(2\vec{a} + \vec{b})^2 = (-\vec{\gamma})^2 \Leftrightarrow$$

$$4|\vec{a}|^2 + 4\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{\gamma}|^2$$

$$4|\vec{a}|^2 + 4\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{\gamma}|^2$$

$$4 \cdot 2^2 + 4\vec{a} \cdot \vec{b} + 1 = 3^2$$

$$16 + 4\vec{a} \cdot \vec{b} + 1 = 9 \Rightarrow 4\vec{a} \cdot \vec{b} = 9 - 1 - 16 \Rightarrow 4\vec{a} \cdot \vec{b} = -8 \Rightarrow \vec{a} \cdot \vec{b} = -2$$

$$(2\vec{a} + \vec{b} + \vec{\gamma})^2 = \vec{0}^2 = 0$$

$$4\vec{a}^2 + \vec{b}^2 + \vec{\gamma}^2 + 4\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{\gamma} + 4\vec{a} \cdot \vec{\gamma} = 0$$

$$X = 2 \Rightarrow$$

$$X^2 = 4$$

$$\vec{a} \cdot \vec{b} = -2$$

Ταυτότητα $(a+b+\gamma)^2 = a^2 + b^2 + \gamma^2 + 2ab + 2a\gamma + 2b\gamma$

$$(a+b+\gamma)^2 = (a+b+\gamma) \cdot (a+b+\gamma) = \underbrace{a^2 + \underline{ab} + \underline{a\gamma} + \underline{ab} + b^2 + \underline{b\gamma} + \underline{a\gamma} + \underline{b\gamma} + \gamma^2}_{= a^2 + b^2 + \gamma^2 + 2ab + 2a\gamma + 2b\gamma}$$

Επιβεβαιώνουμε την πρόταση:

(i) $\vec{a} \cdot \vec{b} = -2$ $\vec{b} \cdot \vec{\gamma} = 1$