

Δίνεται

$$\left. \begin{array}{l} 1 < x \leq 3 \\ 2 \leq y \leq 4 \end{array} \right\} \text{ Να βρεθεί, μεταξύ ποίων τιμών βρίσκεται ο παραγόμενος:}$$

$$A = \frac{1}{x} + \frac{2}{y}, \quad B = x \cdot y, \quad \Gamma = \frac{1}{x \cdot y}.$$

Λύση

Αν a, b οποιεσδήποτε
 $a < b \Leftrightarrow \frac{1}{a} > \frac{1}{b}$

$$1 < x \leq 3 \Leftrightarrow \frac{1}{1} > \frac{1}{x} \geq \frac{1}{3} \Leftrightarrow \frac{1}{3} \leq \frac{1}{x} < 1$$

$$2 \leq y \leq 4 \Leftrightarrow \frac{1}{2} \geq \frac{1}{y} \geq \frac{1}{4} \Leftrightarrow \frac{1}{4} \leq \frac{1}{y} \leq \frac{1}{2}$$

$$\left. \begin{array}{l} 1 < x \leq 3 \\ 2 \leq y \leq 4 \end{array} \right\} 2 < x \cdot y \leq 12 \Leftrightarrow$$

$$\frac{1}{2} > \frac{1}{x \cdot y} \geq \frac{1}{12} \Leftrightarrow \frac{1}{12} \leq \frac{1}{x \cdot y} < \frac{1}{2}$$

$$\frac{1}{3} \leq \frac{1}{x} < 1$$

$$\frac{2}{4} \leq \frac{2}{y} \leq 1 \quad +$$

$$\frac{1}{3} + \frac{1}{2} \leq \frac{1}{x} + \frac{2}{y} < 2 \Leftrightarrow$$

$$\frac{5}{6} \leq \frac{1}{x} + \frac{2}{y} < 2$$

$$\begin{matrix} A^2 \geq 0 \\ B^2 \geq 0 \end{matrix} \left\{ \begin{matrix} (+) \\ \Rightarrow \end{matrix} \right. A^2 + B^2 \geq 0 \quad \Bigg| \quad A^2 + B^2 = 0 \iff A=0 \text{ και } B=0$$

Επίσης $\Sigma - 1. \therefore a^2 + b^2 = 0 \iff a = 0 \text{ και } b = 0$ (1)

$\Sigma \epsilon \lambda. 59-60 / \alpha 6k. 3 - A'OM.$

i) $(x-2)^2 + (y+1)^2 = 0 \iff$

$$\begin{matrix} x-2=0 \Rightarrow \text{και} \\ \textcircled{x=2} \end{matrix} \quad \begin{matrix} y+1=0 \Rightarrow \\ \textcircled{y=-1} \end{matrix}$$

(ii) $x^2 + y^2 - 2x + 4y + 5 = 0$
 $x^2 - 2x + y^2 + 4y + 5 = 0$
 $\underbrace{x^2 - 2 \cdot x \cdot 1 + 1^2}_{(x-1)^2} + \underbrace{y^2 + 2 \cdot y \cdot 2 + 4}_{(y+2)^2} = 0$
 $(x-1)^2 + (y+2)^2 = 0$
 $\begin{matrix} x-1=0 \text{ και} \\ \textcircled{x=1} \end{matrix} \quad \begin{matrix} y+2=0 \\ \textcircled{y=-2} \end{matrix}$

Να βρείτε τα x, y ώστε :

$$1) (x-3)^2 + (x+2y)^2 = 0$$

$$2) 2x^2 + 4y^2 - 6x + 4xy + 9 = 0$$

$$3) (2x-1)^2 + (3y+1)^2 = 0$$

$$4) 4x^2 + 9y^2 - 4x + 6y + 2 = 0$$

$$5) (x+1)^2 + (y+5)^2 = 0$$

$$6) x^2 + y^2 + 2x + 10y + 26 = 0$$