

Να λυθεί η εξίσωση

$$\lambda(\lambda-1)(\lambda-2)x = (\lambda-1)(\lambda+2) \quad (\varepsilon)$$

Η εξίσωση είναι της μορφής

$$ax = b$$

όπου $a = \lambda(\lambda-1)(\lambda-2)$, $b = (\lambda-1)(\lambda+2)$.

{ξέρω} πότε $a = 0 \iff$

$$\lambda(\lambda-1)(\lambda-2) = 0 \iff$$

$\lambda = 0$ ή $\lambda - 1 = 0 \iff$ ή $\lambda - 2 = 0 \iff$

$\lambda = 1$

$\lambda = 2$

1) Αν το $\lambda \neq 0, 1, 2$ τότε $a \neq 0$.

τότε $(\varepsilon) \iff \frac{\lambda(\lambda-1)(\lambda-2) \cdot x}{\lambda(\lambda-1)(\lambda-2)} = \frac{(\lambda-1)(\lambda+2)}{\lambda(\lambda-1)(\lambda-2)}$

$$x = \frac{\lambda+2}{\lambda(\lambda-2)} \quad \text{προσδίκη λύση}$$

2) $\lambda = 0$, $a = 0$, $b = (\lambda-1)(\lambda+2) = (0-1)(0+2) = -1 \cdot 2 = -2$

οπδ $(\varepsilon) \iff 0x = -2$ αδύνατο

3) $\lambda = 1$, $a = 0$, $b = (\lambda-1)(\lambda+2) = (1-1)(1+2) = 0 \cdot 3 = 0$

οπδ $(\varepsilon) \iff 0x = 0$ πανωμωλ.

4) $\lambda = 2$, $a = 0$, $b = (\lambda-1)(\lambda+2) = (2-1)(2+2) = 1 \cdot 4 = 4$

οπδ $(\varepsilon) \iff 0x = 4$, αδύνατο.

4 Διωνυφικές εξισώσεις $X^v = a$

1) $X^2 = 25 \Leftrightarrow X = \pm 5$

2) $X^2 = 7 \Leftrightarrow X = \pm \sqrt{7}$

εξισώσεις $(-\sqrt{7})^2 = \sqrt{7}^2 = 7$.

3) $X^4 = 16 \Leftrightarrow X = \pm 2$

4) $X^4 = 17 \Leftrightarrow X = \pm \sqrt[4]{17}$

5) $X^6 = 25 \Leftrightarrow X = \pm \sqrt[6]{25}$

6) $X^3 = 8 \Leftrightarrow X = 2$

εξισώσεις: $(-2)^3 = -8$

7) $X^5 = 32 \Leftrightarrow X = 2$

8) $X^7 = 15 \Leftrightarrow X = \sqrt[7]{15}$

9) $X^4 = -2$ αδύνατο

10) $X^3 = -8 \Leftrightarrow X = -2$

11) $X^3 = -7 \Leftrightarrow X = -\sqrt[3]{7}$

αδύνατο $\sqrt[3]{-7}$

$X^v = a$

	$a > 0$	$a < 0$
v άρτιος	$X = \pm \sqrt[v]{a}$	αδύνατο
v περιττός	$X = \sqrt[v]{a}$	$X = -\sqrt[v]{ a }$

Πρόβλημα: Να λύσουμε τις εξισώσεις

1) $X^5 - 7X = 0$, 2) $(3x-1)^5 - 7(3x-1) = 0$

Λύση:

1) $X^5 - 7X = 0 \Leftrightarrow$

$X(X^4 - 7) = 0 \Leftrightarrow$

$X = 0$ ή $X^4 - 7 = 0 \Leftrightarrow$

$X^4 = 7 \Leftrightarrow$

$X = \pm \sqrt[4]{7}$

2) Θέτουμε $3x-1 = y$

$y^5 - 7y = 0 \Leftrightarrow$

$y(y^4 - 7) = 0 \Leftrightarrow$

$y = 0$ ή $y = \pm \sqrt[4]{7}$

$3x-1=0$ ή $3x-1 = \pm \sqrt[4]{7}$

$x = \frac{1}{3}$ ή $x = \frac{1 \pm \sqrt[4]{7}}{3}$