

$$2x^2 + 4y^2 - 6x + 4xy + 9 = 0 \Leftrightarrow$$

$$x^2 - 6x + 4y^2 + 4xy + x^2 + 9 = 0 \Leftrightarrow$$

$$x^2 - 2 \cdot x \cdot 3 + 9 + (2y)^2 + 2 \cdot x \cdot 2y + x^2 = 0 \Leftrightarrow$$

$$(x-3)^2 + (2y+x)^2 = 0 \Leftrightarrow$$

$$x-3=0$$

$$x=3$$



$$2y+x=0 \Leftrightarrow$$

$$2y+3=0 \Leftrightarrow$$

$$2y=-3 \Leftrightarrow$$

$$y=-\frac{3}{2}$$

Απόλυτη τιμή
πραγματικοί αριθμοί.

$$|-3|=3, \quad |+5|=5$$



$$|-3|=3 = d(-3, 0)$$

$$|5|=5 = d(5, 0)$$

$$|x| = \begin{cases} x, & \text{αν } x \geq 0 \\ -x, & \text{αν } x < 0 \end{cases}$$

$$|+5| = 5, \quad |-5| = -(-5) = 5$$

8. özümler

$$1) |x| \geq 0, \quad |x| = 0 \Leftrightarrow x = 0$$

$$2) -|x| \leq x \leq |x|$$

$$3) |x|^2 = x^2$$

$$4) |a \cdot b| = |a| \cdot |b|$$

$$5) |a \cdot b \cdot c \cdot d| = |a| \cdot |b| \cdot |c| \cdot |d|$$

$$6) |a^v| = |a|^v$$

$$7) \left| \frac{a}{b} \right| = \frac{|a|}{|b|}, \quad b \neq 0$$

$$8) |a+b| \leq |a| + |b|$$

to "=" if and only if a, b are both non-negative or both non-positive

$$9) |a+b+c+d| \leq |a| + |b| + |c| + |d|$$

$$10) |x| + |y| = 0 \Leftrightarrow x = 0 \text{ and } y = 0$$

$$11) \text{ If } g > 0 \text{ then } |x| < g \Leftrightarrow -g < x < g$$

$$12) \text{ If } g > 0 \text{ then } |x| > g \Leftrightarrow x < -g \text{ or } x > g$$

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Ανοδείξεις: Σχολ. β. βιβλίο 62.

$$(3) \quad |x|^2 = |x| \cdot |x| = \begin{cases} x \cdot x = x^2 & \text{αν } x \geq 0 \\ (-x) \cdot (-x) = x^2 & \text{αν } x < 0 \end{cases}$$

$$\left. \begin{array}{l} |x|^3 = x^3 \text{ αν } x \geq 0 \\ |-2|^3 = 2^3 = 8 \\ (-2)^3 = -8 \end{array} \right\} |x^3| = |x|^3$$

$$(4) \quad |a \cdot b| = |a| \cdot |b| \Leftrightarrow$$

$$|a \cdot b|^2 = (|a| \cdot |b|)^2 \Leftrightarrow$$

$$(a \cdot b)^2 = |a|^2 \cdot |b|^2 \Leftrightarrow$$

$$a^2 \cdot b^2 = a^2 \cdot b^2 \text{ αν } a, b \text{ οποιοδήποτε}$$

$$\text{οπότε } |a \cdot b| = |a| \cdot |b|$$

$$(6) \quad |a^n| = \underbrace{|a \cdot a \cdot a \cdots a|}_{\text{n πολλαπλασιασμοί}} =$$

$$\underbrace{|a| \cdot |a| \cdots |a|}_{\text{n πολλαπλασιασμοί}} = |a|^n$$